Chapter 1

Setting the Scene for Actions in Algebra

In This Chapter

- Enumerating the various number systems
- Becoming acquainted with "algebra-speak"
- Operating on and simplifying expressions
- Converting fractions to decimals and decimals to fractions

What exactly *is* algebra? What is it *really* used for? In a nutshell, *algebra* is a systematic study of numbers and their relationships, using specific rules. You use *variables* (letters representing numbers), and formulas or equations involving those variables, to solve problems. The problems may be practical applications, or they may be puzzles for the pure pleasure of solving them!

In this chapter, I acquaint you with the various number systems. You've seen the numbers before, but I give you some specific names used to refer to them properly. I also tell you how I describe the different processes performed in algebra — I want to use the correct language, so I give you the vocabulary. And, finally, I get very specific about fractions and decimals and show you how to move from one type to the other with ease.

Making Numbers Count

Algebra uses different types of numbers, in different circumstances. The types of numbers are important because what they look like and how they behave can set the scene for particular situations or help to solve particular problems. Sometimes it's really convenient to declare, "I'm only going to look at whole-number answers," because whole numbers do not include fractions or negatives. You could easily end up with a fraction if you're working through a problem that involves a number of cars or people. Who wants half a car or, heaven forbid, a third of a person?

I describe the different types of numbers in the following sections.

Facing reality with reals

Real numbers are just what the name implies: real. Real numbers represent real values — no pretend or make-believe. They cover the gamut and can take on any form — fractions or whole numbers, decimal numbers that go on forever and ever without end, positives and negatives.

Going green with naturals

A *natural number* (also called a *counting number*) is a number that comes naturally. The natural numbers are the numbers starting with 1 and going up by ones: 1, 2, 3, 4, 5, and so on into infinity.

Wholesome whole numbers

Whole numbers aren't a whole lot different from natural numbers (see the preceding section). Whole numbers are just all the natural numbers plus a 0: 0, 1, 2, 3, 4, 5, and so on into infinity.

Integrating integers

Integers are positive and negative whole numbers: . . . –3, –2, –1, 0, 1, 2, 3, ...

Integers are popular in algebra. When you solve a long, complicated problem and come up with an integer, you can be joyous because your answer is probably right. After all, most teachers like answers without fractions.

Behaving with rationals

Rational numbers act rationally because their decimal equivalents behave. The decimal ends somewhere, or it has a repeating pattern to it. That's what constitutes "behaving."

Some rational numbers have decimals that end such as: 3.4, 5.77623, –4.5. Other rational numbers have decimals that repeat the same pattern, such as $3.164164\overline{164}$, or $0.66666666\overline{6}$. The horizontal bar over the 164 and the 6 lets you know that these numbers repeat forever.

TIP

In *all* cases, rational numbers can be written as fractions. Each rational number has a fraction that it's equal to. So one definition of a *rational number* is any number that can be written as a fraction, $\frac{p}{q}$, where p and q are integers (except q can't be 0). If a number can't be written as a fraction, then it isn't a rational number.

Reacting to irrationals

Irrational numbers are just what you may expect from their name — the opposite of rational numbers. An *irrational number* can't be written as a fraction, and decimal values for irrationals never end and never have the same, repeated pattern in them.

Picking out primes and composites

A number is considered to be *prime* if it can be divided evenly only by 1 and by itself. The first prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and so on. The only prime number that's even is 2, the first prime number.

A number is *composite* if it isn't prime — if it can be divided by at least one number other than 1 and itself. So the number 12 is composite because it's divisible by 1, 2, 3, 4, 6, and 12.

Giving Meaning to Words and Symbols

Algebra and symbols in algebra are like a foreign language. They all mean something and can be translated back and forth as needed. Knowing the vocabulary in a foreign language is important — and it's just as important in algebra.

Valuing vocabulary

Using the correct word is so important in mathematics. The correct wording is shorter, more descriptive, and has an exact mathematical meaning. Knowing the correct word or words eliminates misinterpretations and confusion.

- An *expression* is any combination of values and operations that can be used to show how things belong together and compare to one another. An example of an expression is $2x^2 + 4x$.
- A *term,* such as $4xy$, is a grouping together of one or more *factors.* Multiplication is the only thing connecting the number with the variables. Addition and subtraction, on the other hand, separate terms from one another, such as in the expression $3xy + 5x - 6$.
- An *equation* uses a sign to show a relationship — that two things are equal. An example is $2x^2 + 4x = 7$.
- An *operation* is an action performed upon one or two numbers to produce a resulting number. Operations are addition, subtraction, multiplication, division, square roots, and so on.
- A *variable* is a letter representing some unknown; a variable always represents a number, but it varies until it's written in an equation or inequality. (An *inequality* is a comparison of two values.) By convention, mathematicians usually assign letters at the end of the alphabet (such as x, y, and z) to be variables.
- A *constant* is a value or number that never changes in an equation — it's constantly the same. For example, 5 is a constant because it is what it is. By convention, mathematicians usually assign letters at the beginning of the

alphabet (such as *a*, *b*, and *c*) to represent constants. In the equation $ax^2 + bx + c = 0$, *a*, *b*, and *c* are constants and *x* is the variable.

- An *exponent* is a small number written slightly above and to the right of a variable or number, such as the 2 in the expression 3^2. It's used to show repeated multiplication. An exponent is also called the *power* of the value.

Signing up for symbols

The basics of algebra involve symbols. Algebra uses symbols for quantities, operations, relations, or grouping. The symbols are shorthand and are much more efficient than writing out the words or meanings.

- + means *add, find the sum, more than,* or *increased by;* the result of addition is the *sum.* It's also used to indicate a *positive number.*
- – means *subtract, minus, decreased by,* or *less than;* the result is the *difference.* It's also used to indicate a *negative number.*
- × means *multiply* or *times.* The values being multiplied together are the *multipliers* or *factors;* the result is the *product.*

  TIP

  In algebra, the × symbol is used infrequently because it can be confused with the variable *x*. You can use · or * in place of × to eliminate confusion.

  Some other symbols meaning multiply can be grouping symbols: ( ), [ ], { }. The grouping symbols are used when you need to contain many terms or a messy expression. By themselves, the grouping symbols don't mean to multiply, but if you put a value in front of a grouping symbol, it means to multiply. (See the next section for more on grouping symbols.)
- ÷ means divide. The *divisor* divides the *dividend.* The result is the *quotient.* Other signs that indicate division are the fraction line and the slash (/).
- $\sqrt{\ }$ means to take the *square root* of something — to find the number that, multiplied by itself, gives you the number under the sign.

- $|\ |$ means to find the *absolute value* of a number, which is the number itself (if the number is positive) or its distance from 0 on the number line (if the number is negative).
- π is the Greek letter pi, which refers to the irrational number: 3.14159. . . . It represents the relationship between the diameter and circumference of a circle: $\pi = \frac{c}{d}$, where *c* is circumference and *d* is diameter.
- ≈ means *approximately equal* or *about equal.* This symbol is useful when you're rounding a number.

Going for grouping

In algebra, tasks are accomplished in a particular order. After following the order of operations (see Chapter 3), you have to do what's inside a grouping symbol before you can use the result in the rest of the equation.

Grouping symbols tell you that you have to deal with the terms inside the grouping symbols *before* you deal with the larger problem. If the problem contains grouped items, do what's inside a grouping symbol first, and then follow the order of operations. The grouping symbols are

- **Parentheses ():** Parentheses are the most commonly used symbols for grouping.
- **Brackets [ ] and braces { }:** Brackets and braces are also used frequently for grouping and have the same effect as parentheses.

  TIP: Using the different types of grouping symbols helps when there's more than one grouping in a problem. It's easier to tell where a group starts and ends.
- **Radical $\sqrt{\ }$:** This symbol is used for finding roots.
- **Fraction line:** The fraction line also acts as a grouping symbol — everything in the *numerator* (above the line) is grouped together, and everything in the *denominator* (below the line) is grouped together.

Operating with Signed Numbers

The basic operations are addition, subtraction, multiplication, and division. When you're performing those operations on positive numbers, negative numbers, and mixtures of positive and negative numbers, you need to observe some rules, which I outline in this section.

Adding signed numbers

You can add positive numbers to positive numbers, negative numbers to negative numbers, or any combination of positive and negative numbers. Let's start with the easiest situation: when the numbers have the same sign.

There's a nice *S* rule for addition of positives to positives and negatives to negatives. See if you can say it quickly three times in a row: *When the signs are the same, you find the sum, and the sign of the sum is the same as the signs*. This rule holds when *a* and *b* represent any two positive real numbers:

$$(+a) + (+b) = +(a + b) \qquad (-a) + (-b) = -(a + b)$$

Here are some examples of finding the sums of same-signed numbers:

- **(+8) + (+11) = +19:** The signs are all positive.
- **(–14) + (–100) = –114:** The sign of the sum is the same as the signs.
- **(+4) + (+7) + (+2) = +13:** Because all the numbers are positive, add them and make the sum positive, too.
- **(–5) + (–2) + (–3) + (–1) = –11:** This time all the numbers are negative, so add them and give the sum a minus sign.

Numbers with different signs add up very nicely. You just have to know how to do the computation.

When the signs of two numbers are different, forget the signs for a while and find the *difference* between the numbers. This is the difference between their *absolute values.* The number farther from zero determines the sign of the answer:

- $(+a)+(-b)=+(|a|-|b|)$ if the positive a is farther from zero.
- $(+a)+(-b)=-(|b|-|a|)$ if the negative b is farther from zero.

Here are some examples of finding the sums of numbers with different signs:

- **(+6) + (–7) = –1:** The difference between 6 and 7 is 1. Seven is farther from 0 than 6 is, and 7 is negative, so the answer is –1.
- **(–6) + (+7) = +1:** This time the 7 is positive and the 6 is negative. Seven is still farther from 0 than 6 is, and the answer this time is +1.

Subtracting signed numbers

Subtracting signed numbers is really easy to do: You *don't!* Instead of inventing a new set of rules for subtracting signed numbers, mathematicians determined that it's easier to change the subtraction problems to addition problems and use the rules I explain in the previous section. But, to make this business of changing a subtraction problem to an addition problem give you the correct answer, you really change *two* things. (It almost seems to fly in the face of *two wrongs don't make a right,* doesn't it?)

When subtracting signed numbers, change the minus sign to a plus sign *and* change the number that the minus sign was in front of to its opposite. Then just add the numbers using the rules for adding signed numbers:

- $(+a) - (+b) = (+a) + (-b)$
- $(+a) - (-b) = (+a) + (+b)$
- $(-a) - (+b) = (-a) + (-b)$
- $(-a) - (-b) = (-a) + (+b)$

Here are some examples of subtracting signed numbers:

- **–16 – 4 = –16 + (–4) = –20:** The subtraction becomes addition, and the +4 become negative. Then, because you're adding two signed numbers with the same sign, you find the sum and attach their common negative sign.

- **–3 – (–5) = –3 + (+5) = 2:** The subtraction becomes addition, and the –5 becomes positive. When adding numbers with opposite signs, you find their difference. The 2 is positive, because the +5 is farther from 0.
- **9 – (–7) = 9 + (+7) = 16:** The subtraction becomes addition, and the –7 becomes positive. When adding numbers with the same sign, you find their sum. The two numbers are now both positive, so the answer is positive.

Multiplying and dividing signed numbers

Multiplication and division are really the easiest operations to do with signed numbers. As long as you can multiply and divide, the rules are not only simple, but the same for both operations.

When multiplying and dividing two signed numbers, if the two signs are the same, then the result is *positive;* when the two signs are different, then the result is *negative:*

- $(+a) \cdot (+b) = +ab$
- $(+a) \div (+b) = +(a \div b)$
- $(+a) \cdot (-b) = -ab$
- $(+a) \div (-b) = -(a \div b)$
- $(-a) \cdot (+b) = -ab$
- $(-a) \div (+b) = -(a \div b)$
- $(-a) \cdot (-b) = +ab$
- $(-a) \div (-b) = +(a \div b)$

Notice in which cases the answer is positive and in which cases it's negative. You see that it doesn't matter whether the negative sign comes first or second, when you have a positive and a negative. Also, notice that multiplication and division seem to be "as usual" except for the positive and negative signs.

Here are some examples of multiplying and dividing signed numbers:

- $(-8) \cdot (+2) = -16$
- $(-5) \cdot (-11) = +55$
- $(+24) \div (-3) = -8$
- $(-30) \div (-2) = +15$

You can mix up these operations doing several multiplications or divisions or a mixture of each and use the following even-odd rule.

According to the even-odd rule, when multiplying and dividing a bunch of numbers, count the number of negatives to determine the final sign. An *even* number of negatives means the result is *positive.* An *odd* number of negatives means the result is *negative.*

Here are some examples of multiplying and dividing collections of signed numbers:

- **$(+2) \cdot (-3) \cdot (+4) = -24$:** This problem has just one negative sign. Because 1 is an odd number (and often the loneliest number), the answer is negative. The numerical parts (the 2, 3, and 4) get multiplied together and the negative is assigned as its sign.
- **$(+2) \cdot (-3) \cdot (+4) \cdot (-1) = +24$:** Two negative signs mean a positive answer because 2 is an even number.
- **$\frac{(+4)\cdot(-3)}{(-2)} = +6$:** An even number of negatives means you have a positive answer.
- **$\frac{(-12)\cdot(-6)}{(-4)\cdot(+3)} = -6$:** Three negatives yield a negative.

Dealing with Decimals and Fractions

Numbers written as repeating or terminating decimals have fractional equivalents. Some algebraic situations work better with decimals and some with fractions, so you want to be able to pick and choose the one that's best for your situation.

Changing fractions to decimals

All fractions can be changed to decimals. Earlier in this chapter, I tell you that rational numbers have decimals that can be written exactly as fractions. The decimal forms of rational numbers either terminate (end) or repeat in a pattern.

To change a fraction to a decimal, just divide the top by the bottom:

- $\frac{7}{4}$ becomes $4\overline{)7.00} = 1.75$, so $\frac{7}{4} = 1.75$.
- $\frac{4}{11}$ becomes $11\overline{)4.000000\ldots} = 0.363636\ldots$ so $\frac{4}{11} = 0.363636\ldots = 0.\overline{36}$. The division never ends, so the three dots (ellipses) or bar across the top tell you that the pattern repeats forever.

If the division doesn't come out evenly, you can either show the repeating digits or you can stop after a certain number of decimal places and round off.

Changing decimals to fractions

Decimals representing rational numbers come in two varieties: terminating decimals and repeating decimals. When changing from decimals to fractions, you put the digits in the decimal over some other digits and reduce the fraction.

Getting terminal results with terminating decimals

To change a terminating decimal into a fraction, put the digits to the right of the decimal point in the numerator. Put the number 1 in the denominator followed by as many zeros as the numerator has digits. Reduce the fraction if necessary.

Change 0.36 into a fraction:

$$0.36 = \frac{36}{100} = \frac{9}{25}$$

There are two digits in 36, so the 1 in the denominator is followed by two zeros. Both 36 and 100 are divisible by 4, so the fraction reduces.

Change 0.0005 into a fraction:

$$0.0005 = \frac{5}{10{,}000} = \frac{1}{2{,}000}$$

Don't forget to count the zeros in front of the 5 when counting the number of digits. The fraction reduces.

Repeating yourself with repeating decimals

When a decimal repeats itself, you can always find the fraction that corresponds to the decimal. In this chapter, I only cover the decimals that show every digit repeating.

To change a *repeating decimal* (in which every digit is part of the repeated pattern) into its corresponding fraction, write the repeating digits in the numerator of a fraction and, in the denominator, as many nines as there are repeating digits. Reduce the fraction if necessary.

Here are some examples of changing the repeating decimals to fractions:

- $\mathbf{0.126126126\ldots = \frac{126}{999} = \frac{14}{111}}$**:** The three repeating digits are 126. Placing the 126 over a number with three 9s, you reduce by dividing the numerator and denominator by 9.
- $\mathbf{0.857142857142857142\ldots = \frac{857{,}142}{999{,}999} = \frac{6}{7}}$**:** The six repeating digits are put over six nines. Reducing the fraction takes a few divisions. The common factors of the numerator and denominator are 11, 13, 27, and 37.